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Quantifying effect of slope length and steepness on prediction of soil erosion using the Revised Universal Soil Loss Equation (RUSLE) has become very important. In this paper, sequential indicator simulation successfully provided spatial prediction maps of these two variables based on their spatial variability from a plot data set. Uncertainty from simulation runs and semivariograms was studied to increase prediction precision. Uncertainty propagation from slope length and steepness and model parameters to predicting topographical factor LS in RUSLE was modelled using Fourier Amplitude Sensitivity Test (FAST). Spatial variance partitioning was done and uncertainty sources were identified.

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Uncertainty propagation and partitioning in spatial prediction of topographical factor for RUSLE

Key words: Spatial statistics, soil erosion, topographic factors, uncertainty analysis.

Abstract

Quantifying effect of slope length and steepness on prediction of soil erosion using the Revised Universal Soil Loss Equation (RUSLE) has become very important. In this paper, sequential indicator simulation successfully provided spatial prediction maps of these two variables based on their spatial variability from a plot data set. Uncertainty from simulation runs and semivariograms was studied to increase prediction precision. Uncertainty propagation from slope length and steepness and model parameters to predicting topographical factor LS in RUSLE was modelled using Fourier Amplitude Sensitivity Test (FAST). Spatial variance partitioning was done and uncertainty sources were identified.

1. Introduction

To predict the average annual soil loss, the Revised Universal Soil Loss Equation (RUSLE) has been widely used. This equation is a function of rainfall erosivity, soil erodibility, slope length, slope steepness, cover management, and support practice factor (Renard et al., 1997). The slope length factor L and steepness factor S is calculated using slope length and steepness measurements in the field or estimates by a digital elevation model (DEM). The product of L and S, called LS factor, measures the effect of topography on soil erosion. Soil loss arises as slope length and steepness increases and is most sensitive to the LS factor (Benkobi et al., 1994, Biesemans et al., 2000). Wang et al. (2000) also carried out spatial prediction of the LS factor using spatial statistical methods and suggested that sequential indicator simulation was the best method for spatially modeling this factor.

There are several methods for assessing the sources of uncertainty in models. They include Monte Carlo methods, iterated fractional factorial design, Latin hyper-cube sampling, and Taylor series. A good example where numerous sources of uncertainty in model predictions were assessed is by Gertner et al. (1995). They used a second-order Taylor series to partition the uncertainty of projected forest growth into numerous measurement, grouping, sampling and function errors. Another example is the development and application of the Fourier Amplitude Sensitivity Test (FAST) (Cukier et al., 1973, Rodriguez-Camino & Avissar, 1998) for partitioning uncertainty.

Briefly presented in this paper are the results of a case study where spatial prediction of the LS factor using sequential indicator simulation was used, the uncertainty propagation of slope length and steepness to the LS factor was assessed, and techniques to spatially reduce uncertainty were employed. The contributions of these variables and their model parameters to uncertainty of the LS factor were spatially quantified by variance partitioning using FAST.

2. Data set and topographic factors

The study area of 87,890 hectares is located in east Texas. Most slopes are in the 2 to 5 percent range with some slopes that are over 45 percent. The landscape consists of a gently rolling plateau. Surface water drains mostly from west to east. In 1989, 219 field plots were sampled for vegetation and for monitoring soil erosion. Each plot was 100 m by 6m transect. Slope length (λ) in meters was measured as distance of runoff travels between the points of

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origin and deposition, and slope steepness (β) in percent (Tazik et al., 1992). The factors L and S were derived (Moore and Wilson, 1992) as follows:

$$S = \begin{cases} a_1 Sin\beta + a_2 & Tan\beta < 0.09 \\ a_3 Sin\beta - a_4 & Tan\beta \geq 0.09 \\ a_5 Sin^{a_6}\beta + a_7 & \lambda \leq 4m \end{cases} \tag{1}$$

$$L = (\lambda / 22.13)^{(F/(1-F))} \begin{cases} F = (Sin\beta / 0.0896)/(a_5 Sin^{a_6}\beta + a_7) \\ F = 0 \qquad When \quad \lambda \leq 4m \end{cases} \tag{2}$$

3. Simulation and uncertainty analysis

Spatial variability of slope length and steepness for the data set was first derived and fitted using standardization semivariograms and spherical, exponential, Gaussian and power models. The best model was selected. Sequential indicator simulation (Deutsch and Journel, 1998) was then applied to produce prediction and variance maps of the variables. The prediction maps of LS factor was obtained using Eqs. (1) and (2). FAST was used to partition of variance of prediction maps of slope length and steepness. The sources of uncertainty that were accounted for were slope length, steepness, seven parameters in Eqs. (1)-(2), and measurement errors. The components were assumed independent.

Briefly, let a study area be a grid of N nodes, and {Z(**u'**$_j$), j = 1,...,N} be a set of random variables at N locations. The key of sequential indicator simulation is to generate M joint realizations of these N random variables conditional to the data set (Goovaerts, 1997). A continuous variable was first subdivided into K+1 intervals by defining K cutoff values. The data were coded into indicators 0 or 1. A random path visiting nodes was then set. At each node, the K conditional cumulative density function values were determined given the n original data and all simulated values using a kriging method. A value was drawn from the conditional cumulative density function, becoming a conditional datum. This step was repeated until N nodes were visited to obtain a realization. The overall process was repeated M times with possibly different paths, which led to M realizations providing a quantitative measure of spatial uncertainty. The effect from the number of realizations and the number of cutoff values related to number of semivariograms on outputs was evaluated.

Variance contribution of input components was assessed using a partial variance technique based on FAST (Cukier et al., 1973). This was done as part of the sequential indicator simulation on a pixel by pixel bases. Model variance and partial variances contributed by a specific component were computed using Fourier coefficients derived based on the principle of Fourier series and the distribution of components. In our FAST, the distribution of components was determined using different information. The distribution of slope length and steepness at each pixel was then simulated. The distribution of the model parameters was assumed normal distributed. The mean and standard errors of the parameters are listed in Table 1.

Table 1. The model parameters and their distributions

Component	a_1	a_2	a_3	a_4	a_5	a_6	a_7
Mean	10.8	0.03	16.8	0.50	3.0	0.8	0.56
Standard error	1.08	0.003	1.68	0.05	0.3	0.08	0.056

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4. Results

In the left of Fig. 1 are the plot locations and spatial distribution of data for slope steepness, length and LS factor. The steepness values varied from 0 to 56% with most of them less than 5%. Larger values were mainly at the west, southwest and east of the case study area. The length values had a range of 0 to 225m with most of them less than 40m. Larger values were distributed at many small areas such as along the north and northeast boundaries of the case study area. The LS factor has a similar spatial distribution as slope steepness.

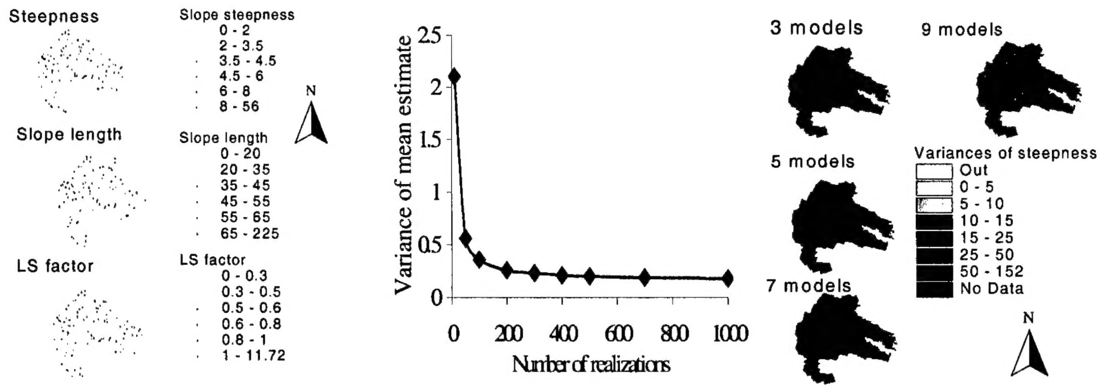

Figure 1, Plot locations and field data (left), simulation variance vs. number of realizations (center), and variance images vs. number of semivariograms used (right).

Table 2 lists the parameters of standardized indicator semivariograms fit using spherical model for slope steepness and length. Seven indicators were obtained, and for each of indicators the probability was calculated and a semivariogram was developed. The semivariograms were used for the simulation.

In the center of Fig. 1 are the variances of mean predicted values for slope steepness. The variance due to simulation became stable after 500 realizations (runs). For all simulations, 500 simulation runs were used. In the right of Fig. 1 are the prediction variance images vs. the number of the used semivariograms. Three, five, seven and nine semivariograms (models) were evaluated. With three and five semivariograms, most of the pixel variances in the images were larger than 40, and the spatial distribution of the variances fairly even for both variance images. With seven and nine semivariograms, the pixel variances decreased rapidly and most of them were less than 15, and the spatial distribution of the variances were not even for both variance images. At the areas with large spatial variability of slope steepness, the prediction variances were high and otherwise low. In addition, the spatial distributions of the uncertainty were similar to that of slope steepness measurements.

Table 2, The modelled omni-directional standardized indicator semivariograms $\gamma(h)$.

$\gamma(h) = c_0 + c_1[1.5\, h/a - 0.5\,(h/a)^3]$ if $h <= a$, and $c_0 + c_1$ otherwise, h=distance, c_0=nugget, c_1=sill, a=range.									
Slope steepness					**Slope length**				
Indicator	Probability	a	C_1	C_0	Indicator	Probability	a	C_1	C_0
0.5	0.046	2982	1.00	0.00	7.0	0.050	2500	0.80	0.20
1.0	0.196	3067	0.95	0.05	10.0	0.137	3000	0.70	0.30
2.0	0.361	2500	0.20	0.80	23.0	0.420	3000	0.20	0.80
3.0	0.580	2877	0.11	0.89	30.0	0.511	3000	0.60	0.40
4.0	0.721	4000	0.15	0.85	40.0	0.694	1835	0.43	0.57

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7.0	0.872	15000	0.00	1.00	70.0	0.849	4000	0.20	0.80
12.0	0.950	4000	0.50	0.50	100.0	0.977	15000	0.00	1.00

The prediction and variance maps of slope steepness and length based on 500 runs and seven semivariograms are shown in the left of Fig. 2. With these maps, the LS prediction map was obtained using Eqs. (1) and (2) and its variance map by modelling the uncertainty propagation from slope steepness and length using the FAST technique. The spatial variability of predicted values were very similar to that of the corresponding variables in the left and centre of Fig. 1. That is, high spatial variability corresponded to large prediction variance.
The relative contribution to uncertainty to prediction of LS due to the parameters, slope length and steepness are show in the right of Fig 2. Overall, the variance contribution from the parameters and slope length were very small, less than 0.03 and 0.22 respectively. Out of the total variance, slope steepness accounted to over 75% of the variability. The data locations had zero contribution because the simulation held data values at the sampling locations.

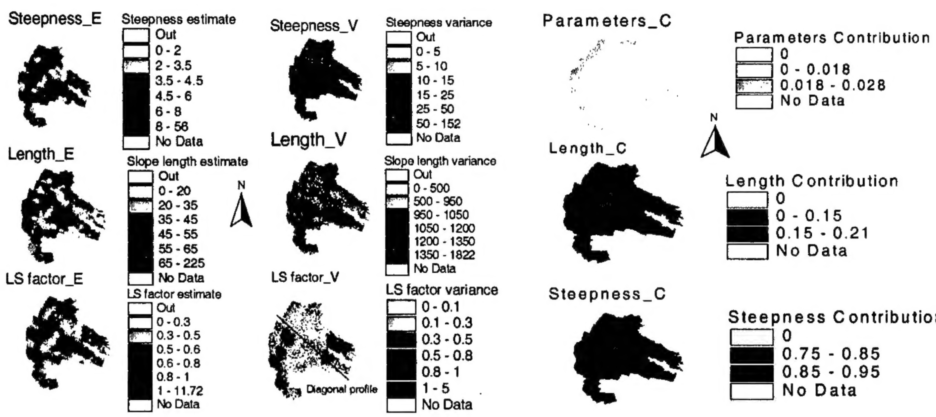

Figure 2, Prediction (left) and variance maps (center) of slope steepness, length and LS factor; and relative variance contributions of input components (right).

In the left of Fig. 3 is diagonal profile across the entire case study area of variance contributions of input components including slope steepness, length and the parameters. The profile had a high peak, implying high uncertainty corresponding to large spatial variability of slope steepness and LS factor at the south-east of the case study area. The variance profile shows that slope steepness resulted in the largest uncertainty, and the slope length and parameters cause a very small part of the uncertainty. The feature is further shown in the right of Fig. 3 for the whole area. The total variance and the variances propagated from the input components increased as the LS estimates (predictions) did. The sensitivity of the LS factor to the components was also analysed using the field data set. The measurement errors of slope steepness and length were evaluated. For example, when measurement errors of slope steepness and length were assumed to be 10% of their means; the percentages in variance contribution from slope steepness, length, measurement errors, and the model parameters respectively are 78.8%, 15.9%, 0.2%, 2.2%, and 2.9%. The variance of LS factor is still mainly from slope steepness.

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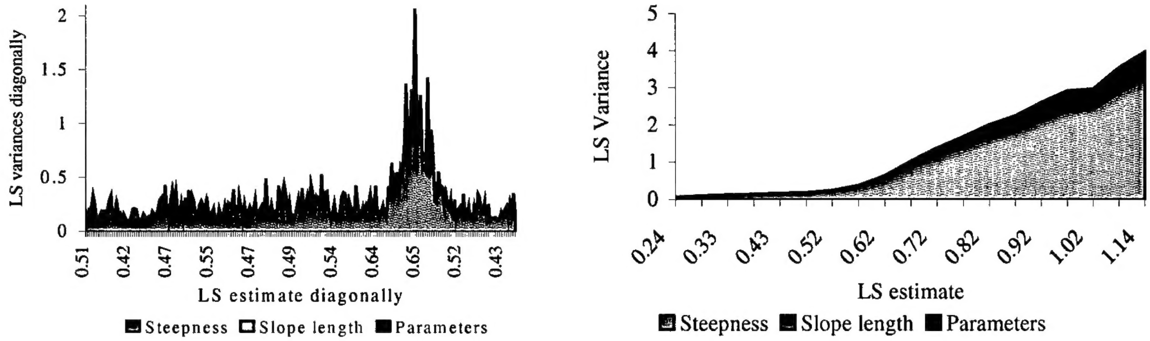

Figure 3, Diagonal profile of variance contributions of the input components (left) and their variance contributions varying with increase of LS estimate for the whole area (right).

5. Conclusion

Sequential indicator simulation successfully produced spatial prediction maps of the slope length, steepness and LS factor with variance images for RUSLE to predict soil loss. Reducing the prediction uncertainty depended greatly on the techniques in simulation. The variance partitioning suggested that the FAST was a powerful tool. The slope steepness contributed the largest uncertainty to prediction of LS factor, then slope length. The contribution by all the model parameters and measurement errors was very small.

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